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Corrections to QCD Born Terms. -Dilepton Production and Leptoproduction

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ABSTRACT

Certain corrections of second order in the Quantum Chromodynamic coupling α_s are studied for dilepton production and for leptoproduction. A problem of convention dependence is analyzed, and an explicit solution is presented. In terms of this it is found that at present dilepton masses M the $0(\alpha_s^2)$ correction to the Drell-Yan formalism due to the subprocess $q+q\rightarrow q+q+\gamma^*$ is small, but that it increases with M. Corresponding corrections to leptoproduction are found to be completely negligible, thus justifying the use of the solution.

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I. INTRODUCTION

During the past year, in the framework of perturbative Quantum Chromodynamics (QCD), the subject of corrections to the Born terms of various processes has attracted much attention.[1-11] Dilepton ($\ell^+\ell^-$) inclusive production is a particular example; then the Born term is determined by the Drell-Yan mechanism

$$q + \bar{q} \rightarrow \overset{*}{\gamma}, \tag{1.1}$$

(q=quark, $\bar{q}$=antiquark, $\overset{*}{\gamma}$=virtual photon $\rightarrow \ell^+\ell^-$) and the QCD corrections of first order in the coupling constant α_s are determined from the subprocesses

$$g + q \rightarrow \overset{*}{\gamma} + q\ , \tag{1.2}$$

(g=gluon) and

$$q + \bar{q} \rightarrow \overset{*}{\gamma} + g\,. \tag{1.3}$$

The perturbation calculation of (1.2) and (1.3) is known to introduce mass singularities. These have been shown to factorize, and can be absorbed by a process independent redefinition of the initial parton distributions.[2] The remaining contribution (of $O(\alpha_s)$) constitutes the correction term.

There is, however, some ambiguity in the process of redefinition. Consider e.g. the subprocess (1.2); the absorption of the corresponding mass singularity proceeds thru the introduction of the density $G_{q/g}$ of a quark in a gluon which is of the form:

$$G_{q/g}(x,q^2) = \frac{\alpha_s}{2\pi}\left[P_{qg}(x)\ln\frac{q^2}{\mu^2} + u_{qg}(x)\right], \tag{1.4}$$

where $q^2 = M^2$ = dilepton mass squared, μ is a regularization mass (see Sec. II) and $P_{qg}(x)$ determines the probability to find a quark in a gluon:[12]

$$P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]. \tag{1.5}$$

In general the function $u_{qg}(x)$ is arbitrary. Thus, although the leading log q^2 contribution is well defined, the nonleading (constant) term is unspecified. This affects the magnitude of the correction term in any phenomenological determination at present.

Concerning the $0(\alpha_s)$ corrections a customary way to remove this ambiguity is the following:[3] One considers leptoproduction, where the Born term corresponds to

$$\gamma^* + q \to q \tag{1.6}$$

and the $0(\alpha_s)$ contributions come from the subprocesses

$$\gamma^* + g \to q + \bar{q}, \tag{1.7}$$

and

$$\gamma^* + q \to g + q; \tag{1.8}$$

then one requires that leptoproduction is free of $0(\alpha_s)$ corrections. This fixes $u_{qg}(x)$ and a similar function associated with (1.3) and (1.8).

Recently the $0(\alpha_s^2)$ correction to the Drell-Yan formalism from the subprocess

$$q + q \rightarrow \overset{*}{\gamma} + q + q , \tag{1.9}$$

(q-q correction, Fig. 1a) has also been studied.[10] Because of the presence of valence quarks in the initial state, this correction was anticipated to be large.[1] The subprocess (1.9) introduces two new density functions ($G_{g/q}$ and $G_{\bar{q}/q}$, see Ref. 10 and Sec. 2) and a new two-fold ambiguity. As a result the above procedure through leptoproduction[3] does not remove the ambiguity.

In the present work we propose an approach that solves this problem. For dilepton production we completely specify the $0(\alpha_s^2)$ correction term due to (1.9). Our conclusion is that, at presently available $M_{\ell^+\ell^-}$ this correction is small, but that it increases as $M_{\ell^+\ell^-}$ approaches the kinematic limit ($\rightarrow \sqrt{s}$). Furthermore in leptoproduction our approach determines the $0(\alpha_s^2)$ correction due to

$$\overset{*}{\gamma} + q \rightarrow q + q + \bar{q} , \tag{1.10}$$

(Fig. 1b). This is found to be completely negligible.

Section II presents our formalism and the essential results of the perturbation calculation of (1.9); also it states in a clear way the problem of the $0(\alpha_s^2)$ ambiguities. Section III contains our approach and our explicit solution. Section IV applies the solution to the determination of the above $0(\alpha_s^2)$ corrections to dilepton production and to leptoproduction. Section V compares our results with certain other conventions. Section VI presents our conclusions.

We note that (1.9) provides also a correction to the QCD Born terms that determine the transverse momentum distribution of dileptons in $p+p \to \ell^+\ell^- + X$ and the direct photon production at large transverse momenta in $p+p \to \gamma + X$; the Born term contribution to these processes is known to be dominated by (1.2). Our approach completely specifies these corrections, as well. We intend to report on this in a subsequent publication.

II. BASIC FORMALISM

We begin by briefly considering the contribution of (1.2) to dilepton production and of the corresponding (1.7) to leptoproduction. Our regularization procedure consists of taking the gluon slightly off-shell ($p^2 = -\mu^2 < 0$). Then the perturbation calculation of (1.2) gives in the Feynman gauge[3,6]

$$\frac{d\sigma_{qg}}{dq^2} = \frac{4\pi\alpha_{em}^2}{3sq^2}\,\frac{e_q^2}{4N}\,\frac{\alpha_s}{\pi}\left\{2P_{qg}(\tau)\ln\frac{q^2(1-\tau)}{-p^2\tau^2} + \frac{(1-\tau)^2}{2} + 2\tau(1-\tau)-1\right\}, \tag{2.1}$$

where e_q is the fractional quark charge, N=3 for color SU(3), α_{em}=1/137 and

$$\tau \equiv \frac{q^2}{s}\ . \tag{2.2}$$

Likewise the $0(\alpha_s)$ contribution of (1.7) to the leptoproduction structure function is

$$\frac{1}{x} F_2(x, q^2) = \frac{\alpha_s}{\pi} e_q^2 \, 2 \left\{ P_{qg}(x) \ln \frac{q^2}{p^2 x^2} + 3x(1-x) - 1 \right\} . \tag{2.3}$$

We require no correction term of $0(\alpha_s)$ from the subprocess (1.7). This requirement implies the form (1.4) for the density $G_{q/g}(x, q^2)$ with

$$u_{qg}(x) = -2 P_{qg}(x) \ln x + 3x(1-x) - 1 . \tag{2.4}$$

Throughout the present work we keep this convention which completely fixes $G_{q/g}$. Then the subprocess (1.2) contributes to dilepton production a correction term proportional to:

$$\kappa^{(1)}_{gq}(\tau) = \frac{\alpha_s}{\pi} \left\{ P_{qg}(\tau) \ln(1-\tau) + \frac{3}{4} - \frac{5}{2} \tau + \frac{9}{4} \tau^2 \right\} . \tag{2.5}$$

We turn now to the contribution of the subprocess (1.9) which is the main subject of this paper. The result of the perturbation calculation[10,13] can be cast in the form:

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3sq^2} \frac{e_q^2}{2N} \left(\frac{\alpha_s}{2\pi}\right)^2 \int_\tau^1 \frac{d\alpha}{\alpha} F(\alpha, \tau; q^2) , \tag{2.6}$$

with the definition:

$$P_{gq}(x) = C_2 \frac{1 + (1-\alpha)^2}{\alpha} ,$$

$(C_2 \equiv (N^2-1)/2N)$, $F(\alpha,\tau;q^2)$ contains the following terms[14]

$$F(\alpha,\tau;q^2) = \Lambda_2(\alpha,\frac{\tau}{\alpha})\ln^2\frac{q^2}{-p^2} + \Lambda_1(\alpha,\frac{\tau}{\alpha})\ln\frac{q^2}{-p^2} + \Lambda_0(\alpha,\frac{\tau}{\alpha}), \tag{2.7}$$

where:

$$\Lambda_2(\alpha,z) = P_{gq}(\alpha)\,P_{qg}(z), \tag{2.8}$$

$$\Lambda_1(\alpha,z) = P_{gq}(\alpha)\left[2P_{qg}(z)\left(\ln\frac{1-z}{z}-1\right)+\frac{1}{2}+z-\frac{3}{2}z^2\right] + \\ + 8C_2\frac{1-\alpha}{\alpha}z(1-z) - 2C_2\frac{2-\alpha}{\alpha}P_{qg}(z), \tag{2.9}$$

$$\Lambda_0(\alpha,z) \simeq P_{gq}(\alpha)\left\{P_{qg}(z)\left[\ln^2(1-z)-2\ln z\ln(1-z)+2\int_0^{1-z}\frac{d\beta}{\beta}\ln(1-\beta)\right]+\right. \\ \left.+\frac{1-z}{z}(1+3z)\ln(1-z)+\frac{3}{2}z^2\ln z-\frac{z}{2}(1-z)\right\}+ \\ +2[1-\ln(1-z)]\left\{P_{gq}(\alpha)\,P_{qg}(z)-4C_2\frac{1-\alpha}{\alpha}z(1-z)\right\} \\ -2C_2\frac{2-\alpha}{\alpha}\left\{P_{qg}(z)\left(-2+\ln\frac{1-z}{z}\right)+\frac{1-z}{4}(1+3z)\right\}. \tag{2.10}$$

Equation (2.6) can be rewritten in the following general form:

$$\frac{d\sigma_{qq}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3sq^2}\left\{\int_\tau^1\frac{dx}{x}\left[G_{\bar{q}/q_1}(x,q^2)\kappa^{(0)}_{\bar{q}q_2}\left(\frac{\tau}{x}\right)+G_{g/q_1}(x,q^2)\kappa^{(1)}_{gq_2}\left(\frac{\tau}{x}\right)+\right.\right. \\ \left.\left.+(q_1\leftrightarrow q_2)\right]+\kappa^{(2)}_{q_1q_2}(\tau)\right\}, \tag{2.11}$$

$\kappa^{(1)}_{gq}(\tau)$ is the correction term of (2.5) and

$$\kappa^{(0)}_{\bar{q}q}(\tau) = \frac{e_q^2}{2N}\,\delta(1-\tau)\,. \tag{2.12}$$

$G_{g/q}$ ($G_{\bar{q}/q}$) is the gluon (antiquark) density in a quark, with the form

$$G_{g/q}(x,q^2) = \frac{\alpha_s}{2\pi}\left\{P_{gq}(x)\,\ln\frac{q^2}{-p^2} + u_{gq}(x)\right\}, \tag{2.13}$$

$$G_{\bar{q}/q}(x,q^2) = \left(\frac{\alpha_s}{2\pi}\right)^2 \int_x^1 \frac{d\alpha}{\alpha}\left\{P_{\bar{q}g}\left(\frac{x}{\alpha}\right) P_{gq}(\alpha)\ln^2\frac{q^2}{-p^2} + K(x,\alpha)\ln\frac{q^2}{-p^2} + \right.$$
$$\left. + u_{\bar{q}q}(x,\alpha)\right\}, \tag{2.14}$$

where

$$K(x,\alpha) = P_{gq}(\alpha)\left[2P_{qg}\left(\frac{\tau}{\alpha}\right)\left(\ln\frac{\alpha}{\tau^2} - 1\right) + 6\,\frac{\tau}{\alpha}\left(1-\frac{\tau}{\alpha}\right) - 1\right] + 8C_2\,\frac{1-\alpha}{\alpha}\,\frac{\tau}{\alpha}\left(1-\frac{\tau}{\alpha}\right)$$
$$-2C_2\,P_{qg}\left(\frac{\tau}{\alpha}\right)\frac{2-\alpha}{\alpha}\,. \tag{2.15}$$

The functions $u_{gq}(x)$ and $u_{\bar{q}q}(x,\alpha)$ are the subject of the subsequent determination. Finally $\kappa^{(2)}_{qq}(\tau)$ is the q-q correction to dilepton production; it is easily determined in terms of (2.10) and of $u_{gq}(x)$ and $u_{\bar{q}q}(x,\alpha)$. We note that the possibility of writing Eq. (2.7) in the form (2.11) follows from general considerations.

It is now evident that, unless both $u_{gq}(x)$ and $u_{\bar{q}q}(x,\alpha)$ are specified, the correction $\kappa^{(2)}_{qq}(\tau)$ remains undetermined. At first sight, an attractive way appears to be the extension to $0(\alpha_s^2)$ of the $0(\alpha_s)$ approach of Ref. 3:

Calculate the contribution to the leptoproduction structure function of the subprocess (1.10) (Fig. 1b); this can be written in the general form

$$\frac{1}{x} F_2(x,q^2) = \int_x^1 \frac{dy}{y}\left[G_{\bar{q}/q}(y,-q^2)\, C^{(0)}_{\overset{*}{\gamma}\bar{q}}\left(\frac{x}{y}\right) + G_{g/q}(y,-q^2)\, C^{(1)}_{\overset{*}{\gamma}g}\left(\frac{x}{y}\right)\right] + $$

$$+ C^{(2)}_{\overset{*}{\gamma}q}(x) , \tag{2.16}$$

where

$$C^{(0)}_{\overset{*}{\gamma}\bar{q}}(x) = e_q^2\, \delta(1-x) ,$$

$C^{(1)}_{\overset{*}{\gamma}g}$ is the $0(\alpha_s)$ correction from (1.7) and $C^{(2)}_{\overset{*}{\gamma}q}(x)$ the $0(\alpha_s^2)$ correction from (1.10). Then impose the requirement $C^{(2)}_{\overset{*}{\gamma}q}(x) \equiv 0$.

Unfortunately this approach specifies only the function $u_{\bar{q}q}(x,\alpha)$. The requirement that leptoproduction is free of $0(\alpha_s)$ correction from (1.7) actually means $C^{(1)}_{\overset{*}{\gamma}g}(x/y) \equiv 0$. Thus the density $G_{g/q}$ does not enter Eq. (2.16) and the function $u_{gq}(x)$ remains completely unspecified.

III. DETERMINATION OF PARTON DENSITIES

We proceed in the specification of the functions $u_{gq}(x)$, $u_{\bar{q}q}(x,\alpha)$ and in the complete determination of the parton densities $G_{g/q}(x,q^2)$ and $G_{\bar{q}q}(x,q^2)$.

Denote by $G_{g/h}(x)$ the gluon distribution in the hadron h. Since gluons emit antiquarks with a finite probability ($\sim P_{\bar{q}/g}(x) = P_{q/g}(x)$), $G_{g/h}$ gives the following $0(\alpha_s)$ contribution to the $\bar{q}$ distribution in h:[12]

$$G^{(1)}_{\bar{q}/h}(x, q^2) = \int_x^1 \frac{d\alpha}{\alpha} G_{\bar{q}/g}\left(\frac{x}{\alpha}, q^2\right) G_{g/h}(\alpha) . \tag{3.1}$$

This is a basic relation employed in the absorption of mass singularities by a redefinition of the parton distributions.

Consider the extension of this relation to the case $h \equiv$ quark. Then we obtain the following $0(\alpha_s^2)$ condition between parton densities:

$$G_{\bar{q}/q}(x, q^2) = \int_x^1 \frac{d\alpha}{\alpha} G_{\bar{q}/g}\left(\frac{x}{\alpha}, q^2\right) G_{g/q}(\alpha, q^2) . \tag{3.2}$$

Replacing the expressions (2.4), (2.15) and (2.16) we see that with respect to terms of $0(\ln^2 q^2/-p^2)$ the condition (3.2) is satisfied automatically. This suggests that we require (3.2) to hold for the complete densities $G_{\bar{q}/q}$, $G_{\bar{q}/g}$ and $G_{g/q}$. With this requirement equality of the terms of $0(\ln q^2/-p^2)$ and of $0(1)$ implies correspondingly:

$$\int_x^1 \frac{d\alpha}{\alpha} \left[u_{qg}\left(\frac{x}{\alpha}\right) P_{gq}(\alpha) + P_{qg}\left(\frac{x}{\alpha}\right) u_{gq}(\alpha)\right] = \int_x^1 \frac{d\alpha}{\alpha} K(x, \alpha), \tag{3.3}$$

and

$$u_{\bar{q}q}(x, \alpha) = u_{qg}\left(\frac{x}{\alpha}\right) u_{gq}(\alpha) . \tag{3.4}$$

Equation (3.3) is an integral equation for the unknown function $u_{gq}(\alpha)$; we will see that it leads to a unique solution. Then Eq. (3.4) determines uniquely the remaining function $u_{\bar{q}q}(x, \alpha)$. Thus the condition (3.2) leads to a complete determination of the densities $G_{g/q}$ and $G_{\bar{q}/q}$ and of the correction $\kappa^{(2)}_{qq}(\tau)$.

To solve the integral Eq. (3.3) we introduce the moments $f(n)$ of the function $f(x)$:

$$f(n) \equiv \int_0^1 dx\, x^{n-1} f(x) \,, \tag{3.5}$$

and make use of the convolution theorem for Mellin transform

$$f(x) = \int_x^1 \frac{d\alpha}{\alpha}\, u(\alpha)\, v\left(\frac{x}{\alpha}\right), \tag{3.6a}$$

$$f(n) = u(n)\, v(n) \,. \tag{3.6b}$$

Denoting by $k(n)$ the n-th moment of the right-hand side of Eq. (3.3) we obtain:

$$u_{gq}(n) = \frac{1}{P_{qg}(n)} \left[k(n) - u_{qg}(n)\, P_{gq}(n)\right] \,. \tag{3.7}$$

Straightforward (but lengthy) calculation gives after use of inverse Mellin transforms and proper analytic continuation in n:

$$u_{gq}(x) = 2\,C_2 \left\{ -\frac{2}{3}\, x^2 - x + \frac{1}{6x} - 2\, \frac{(1-x)^2}{x} \ln x \right.$$
$$\left. - \frac{\sqrt{x}}{2} \left[\cos(\theta_0 \ln x) - \frac{3}{2\theta_0} \sin(\theta_0 \ln x)\right]\right\}, \tag{3.8}$$

where $\theta_0 = \sqrt{7}/2$. This expression uniquely specifies the function $u_{gq}(x)$.

We remark that as $x \to 0$ the last two terms of (3.8) ($\sim \sqrt{x}\cos(\theta_0 \ln x)$ and $\sim \sqrt{x}\sin(\theta_0 \ln x)$) oscillate with a decreasing amplitude. However, in the contribution to the physical process $h_1 + h_2 \to \ell^+ \ell^- + X$ the function (3.8) is multiplied by the quark distributions in the hadrons h_1, h_2 and integrated (see below Eqs. (4.1) and (4.3)). This integration smears the contribution and eliminates the oscillations.

IV. $0(\alpha_s^2)$ CORRECTIONS TO DILEPTON PRODUCTION AND LEPTOPRODUCTION

The inclusive cross-section for the process $h_1 + h_2 \to \ell^+\ell^- + X$ receives the following contribution from $\kappa^{(2)}_{q_1 q_2}$:

$$\frac{d\sigma^{(qq)}_{h_1 h_2}}{dq^2} = \frac{4\pi\alpha^2_{em}}{3sq^2} \sum_{q_1, q_2} \iint \frac{dx_1}{x_1} \frac{dx_2}{x_2} G_{q_1/h_1}(x_1, q^2) G_{q_2/h_2}(x_2, q^2) \kappa^{(2)}_{q_1 q_2}(\tau_{12}), \tag{4.1}$$

where $\tau_{12} \equiv \tau/x_1 x_2$ and $G_{q/h}$ is the distribution of the quark q in the hadron h. As discussed in Refs. 10 the dominant (by far) contribution of $\kappa^{(2)}_{q_1 q_2}(\tau_{12})$ comes from the region $\tau_{12} \simeq 1$. This because for x_1 and/or $x_2 \to 1$ the distribution $G_{q_1/h}$ and/or G_{q_2/h_2} decreases fast. Thus only the leading terms for $\tau_{12} \to 1$ need be considered.

The general form of $\kappa^{(2)}_{q_1 q_2}(\tau)$ is:

$$\kappa^{(2)}_{q_1 q_2}(\tau) = \left(\frac{\alpha_s}{\pi}\right)^2 \left[\left(e^2_{q_1} + e^2_{q_2}\right) C_A(\tau) + 2 e_{q_1} e_{q_2} C_B(\tau)\right]. \tag{4.2}$$

In the limit $\tau \to 1$ the term $C_B(\tau)$ gives only nonleading contributions[10,13] and will be subsequently discarded; anyway its contribution is given in Refs. 10 and 13. Then in terms of (2.9)-(2.11), (3.8) and (3.4):

$$C_A(\tau) \simeq \frac{1}{8N}\int_\tau^1 \frac{d\alpha}{\alpha}\Big\{\Lambda_2(\alpha,\frac{\tau}{\alpha})\ \ln^2\tau - \Lambda_1(\alpha,\frac{\tau}{\alpha})\ \ln^2\tau + \Lambda_0(\alpha,\frac{\tau}{\alpha})$$

$$- \left[2\,P_{qg}\left(\frac{\tau}{\alpha}\right)\ \ln(1-\frac{\tau}{\alpha}) + \frac{3}{2} - 5\,\frac{\tau}{\alpha} + \frac{9}{2}\,\frac{\tau^2}{\alpha^2}\right] u_{gq}(\alpha) - u_{\overline{q}q}(\tau,\alpha)\Big\}\ . \tag{4.3}$$

Using this expression with $u_{gq}(\alpha)$ given by (3.8) and $u_{\overline{q}q}(\tau,\alpha)$ by (3.4) we have calculated (4.1) for $p+p\to\ell^+\ell^-+X$. We use the distributions $G_{q/h}(x,q^2)$ of Ref. 15 (counting-rule like solution). In Fig. 2 (solid lines) we present our results for the ratio

$$R(\tau,s) = \frac{d\sigma^{(qq)}_{pp}}{dq^2}\Bigg/\left(\frac{d\sigma_{pp}}{dq^2}\right)_{DY}\ , \tag{4.4}$$

where $(d\sigma_{pp}/dq^2)_{DY}$ is the Drell-Yan cross-section calculated with the q^2-dependent distributions of Ref. 15.

As $\tau\to 1$ the limiting form of $C_A(\tau)$ is

$$C_A(\tau) \simeq \frac{C_2}{8N}\ (1-\tau)\left[\frac{1}{2}\ \ln^2(1-\tau) + \ln(1-\tau) + 2\right]. \tag{4.5}$$

We have calculated $R(\tau,s)$ using also this form (Fig. 2, dashed lines); for all $\tau \gtrsim 0.3$ the difference from the previous calculation is insignificant. Clearly, for all physical purposes the approximation (4.5) is sufficient.

The basic conclusion is that, at presently available dilepton masses ($\tau \lesssim 0.3$) the q-q correction to dilepton production, calculated with the functions (3.8) and (3.4), is very small. This is true at Brookhaven ($\sqrt{s} \simeq 6.5$ GeV) and even more at Fermilab energies ($\sqrt{s} \simeq 27$ GeV), and conforms with the fact that the Drell-Yan mechanism explains all the basic features of the data.[16,17]

As τ increases the correction becomes more important. This is mainly due to the presence of a $\bar{q}$ distribution in $(d\sigma_{pp}/dq^2)_{DY}$, as opposed to $d\sigma_{pp}^{(qq)}/dq^2$ which contains valence q.[10] Unfortunately manifestations of this correction[10] cannot be easily detected, because at large τ the cross-sections are very small and hard to measure and analyze.

Now we turn to the contribution of $\overset{*}{\gamma}+q \to q+q+\bar{q}$ (Fig. 16) to the structure function $F_2(x, q^2)$ of leptoproduction. Direct perturbation calculation leads to the expression (2.18) with the density $G_{\bar{q}/q}(x, q^2)$ given by (2.16) and with

$$C^{(2)}_{\overset{*}{\gamma}q}(x) = e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \left\{\phi(x) - \int_x^1 \frac{d\alpha}{\alpha}\, u_{\bar{q}q}(x, \alpha)\right\}; \qquad (4.6)$$

in the limit $x \to 1$ (which suffices for our purpose):

$$\phi(x) \simeq C_2 \left(\frac{7\pi^2}{24} + 2\right)(1 - x) . \qquad (4.7)$$

Using (3.8) and (3.4) we obtain for not too small x:

$$C^{(2)}_{\overset{*}{\gamma}q}(x) \simeq e_q^2 \left(\frac{\alpha_s}{2\pi}\right)^2 C_2 \left(\frac{7\pi^2}{24} - 2\right)(1 - x) . \qquad (4.8)$$

At $x \simeq 0.6$ and $-q^2 \approx 10\ \mathrm{GeV}^2$ this implies ~2% correction in the structure function. Since in our approach $C^{(1)}_{\overset{*}{\gamma}g}(x) \equiv 0$ we may conclude that both subprocesses $\overset{*}{\gamma} + g \to q + \bar{q}$ and $\overset{*}{\gamma} + q \to q + q + \bar{q}$ leave leptoproduction practically unaffected.

V. COMPARISON WITH OTHER CONVENTIONS

The question of the magnitude of the q-q correction to dilepton production was first studied in Refs. 10. Those works define the densities $G_{g/q}(x, q^2)$ and $G_{\bar{q}/q}(x, q^2)$ in a somewhat different way. The convention of the first of Refs. 10 amounts to:

$$u_{gq}(x) = - P_{gq}(x) \ln x - C_2 \frac{2-x}{x} ; \qquad (5.1)$$

also $G_{\bar{q}/q}(x, q^2)$ somewhat differs from (2.14). As a result, for $\tau \to 1$ the limiting form of $C_A(\tau)$ is[10]

$$C_A(\tau) \simeq \frac{C_2}{8N} (1-\tau) \left[\frac{1}{2} \ln^2 (1-\tau) - 2 \ln (1-\tau) + 3 \right] . \qquad (5.2)$$

In Fig. 2 we present (dash-dotted lines) the calculation of $R(\tau, s)$ with (5.2). The resulting correction is somewhat bigger, but of the same order of magnitude for all τ of interest. We may conclude that the conventions of Ref. 10, although somewhat arbitrary, give a q-q correction to the Drell-Yan formalism of acceptable (and plausible) magnitude.

In a recent paper[11] a different convention is proposed. To see its origin we introduce also the quark density in a quark:

$$G_{q/q}(x,q^2) = \frac{\alpha_s}{2\pi}\left[P_{qq}(x)\ln\frac{q^2}{-p^2} + u_{qq}(x)\right]. \tag{5.3}$$

Here[12]

$$P_{qq}(x) = C_2\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(x-1)\right], \tag{5.4}$$

where the distribution $1/(1-x)_+$ is defined by the relation:

$$\int_0^1 \frac{dx\, f(x)}{(1-x)_+} \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} . \tag{5.5}$$

The function $u_{qq}(x)$ is specified by requiring[3,11] that to $0(\alpha_s)$ the sub-process (1.8) give no correction to leptoproduction. This implies

$$u_{qq}(x) = C_2\left[-2\,\frac{1+x^2}{1-x}\,\ln x + 1 + 3x - \frac{3}{2}\,\frac{1}{(1-x)_+} - 2\,\frac{\pi^2}{3}\,\delta(1-x)\right]. \tag{5.6}$$

Reference 11 considers the conservation of momentum sum-rule by the gluon field. This implies the following condition between the second moments $(n=2)$ of the functions $u_{gq}(x)$ and $u_{qq}(x)$:

$$u_{gq}(n=2) = -\,u_{qq}(n=2) . \tag{5.7}$$

Then as a possible complete definition of the gluon field, Ref. 11 proposes the extension of the condition (5.7) to all moments n. This leads to the specification

$$u_{gq}(x) = -\,u_{qq}(x) . \tag{5.8}$$

We shall determine the correction term $\kappa^{(2)}_{q_1q_2}(\tau)$ and the corresponding $d\sigma^{(qq)}_{pp}/dq^2$ implied by the convention (5.8) together with (5.6). We need to specify $u_{\bar{q}q}(x)$ as well, and for this we require no $0(\alpha_s^2)$ correction from (1.10) (see end of Sec. 2):

$$C^{(2)}_{\overset{*}{\gamma}q}(x) \equiv 0 \ . \tag{5.9}$$

Now for τ not too small $C_A(\tau)$ has the form:

$$C_A(\tau) \simeq \frac{C_2}{8N}\left\{\frac{3}{2}\ln^2(1-\tau) - \frac{2\pi^2}{3}\ln(1-\tau) - \frac{5}{12}\pi^2 + \right.$$
$$\left. + (1-\tau)\left[-\frac{5}{2}\ln^2(1-\tau) + \left(\frac{1}{2}+\frac{4}{3}\pi^2\right)\ln(1-\tau)\right]\right\} . \tag{5.10}$$

Figure 2 shows the resulting $R(\tau, s)$ for $\sqrt{s} = 27$ GeV (dash-dot-dotted line; notice the result is multiplied by 10^{-2}). Clearly the convention (5.8) introduces too large a correction to dilepton production for almost all τ. It is not difficult to see the reason: As $\tau \to 1$, in both (4.5) and (5.2) $C_A(\tau) \to 0$; however in (5.10) $C_A(\tau) \to \infty$, and this is due to the singular terms $\sim 1/(1-x)_+$ and $\sim \delta(1-x)$ in (5.6). Since it is the behaviour near $\tau_{12} = 1$ that dominates the integral of (4.1), Eq. (5.10) leads to a correction exceeding by two orders of magnitude those of (4.5) and (5.2).

Such a large correction would render useless all the successful phenomenology of dilepton production based on the Drell-Yan formalism. We may conclude that extension of (5.7) to all moments is a condition too strong (and unnessary).

One may ask whether $u_{gq}(x)$ of Eq. (3.8) satisfies the condition (5.7). We find that, with $u_{qq}(x)$ chosen as in (5.6), Eq. (5.7) is not satisfied. However this is, probably, not a serious defect of (3.8). As we repeatedly stated in this paper and elsewhere,[10] what really counts for our purpose is the behaviour of $u_{gq}(x)$ near $x = 1$; near $x = 0$, $u_{gq}(x)$ can be modified with no significant effect. Even if one admits (5.6), the condition (5.7), being a requirement for the lowest moment, can be satisfied by changing (3.8) only near $x = 0$.[18]

VI. CONCLUSIONS

We have seen that the condition (3.2), which is anyway satisfied for the leading logarithms (of $0(\log^2 q^2/p^2)$), if it is required to hold for the nonleading terms as well, it completely specifies the parton densities $G_{g/q}$ and $G_{\bar{q}/q}$. This leads to an unambiguous determination of the q-q correction (subprocess (1.9)) to dilepton production; at present energies and dilepton masses ($\tau \lesssim 0.3$) the correction is found to be small, thus leaving unspoiled the successes of the Drell-Yan formalism. It also leads to a well-specified $0(\alpha_s^2)$ correction to the leptoproduction structure function $F_2(x, q^2)$, due to the subprocess (1.10); this correction is found to be completely negligible, thus leaving unaffected the traditional procedure of determining quark distributions via leptoproduction data analyses.

The density $G_{g/q}(x, q^2)$ also controls the corrections to QCD Born terms (subprocess (1.2)) determining the transverse momentum (p_T)

distribution of $\ell^+\ell^-$ in $p+p \to \ell^+\ell^- + X$ and of γ in $p+p \to \gamma + X$. In a subsequent paper we show that our $G_{g/q}$ implies an important correction to both these processes at large p_T.

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FIGURE CAPTIONS

Fig. 1: Typical Feynman graphs determining the $O(\alpha_s^2)$ contributions of the subprocesses

(1a) $q + q \to \overset{*}{\gamma} + q + q$ to dilepton production,

(1b) $\overset{*}{\gamma} + q \to q + q + \bar{q}$ to leptroproduction.

Fig. 2: The ratio $R(\tau, s) \equiv (d\sigma^{(qq)}_{pp}/dq^2)/(d\sigma_{pp}/dq^2)_{DY}$. Solid lines correspond to Eq. (4.3); dashed lines to Eq. (4.5); dashed-dotted lines to Eq. (5.2); dash-dot-dotted line to Eqs. (5.8) and (5.6) (only for $\sqrt{s} = 27$ GeV; notice here the result is multiplied by 10^{-2}).

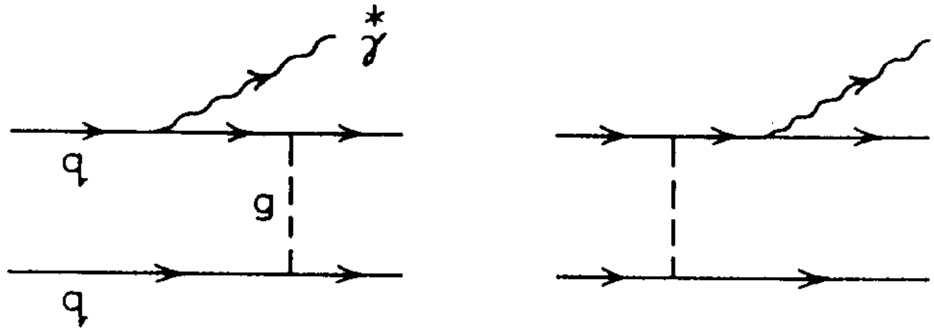

Fig. 1a

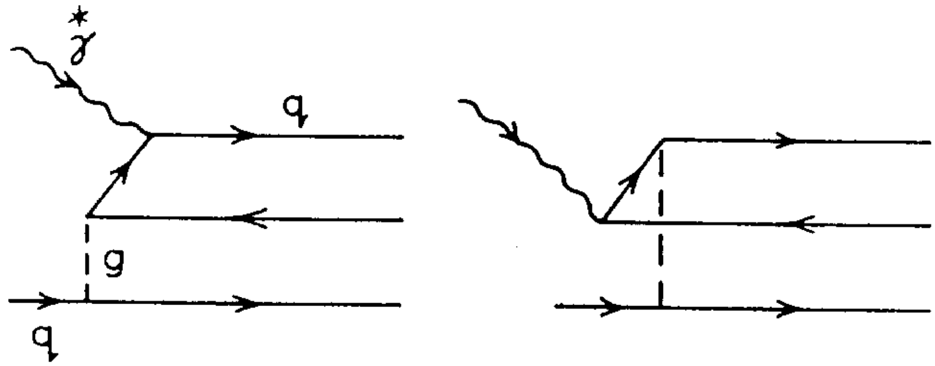

Fig. 1b

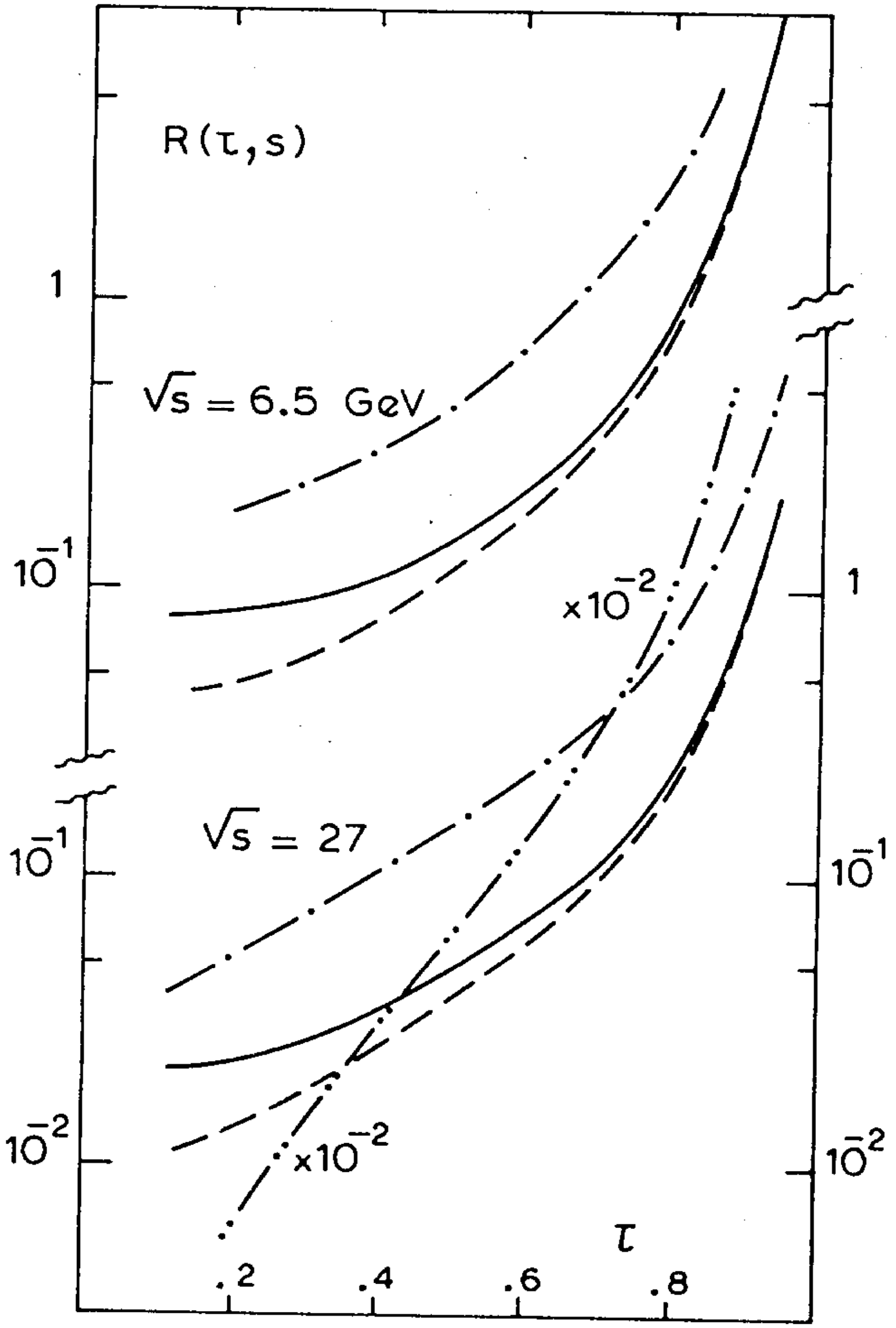

Fig. 2